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SEMI WEAKLY COMPATIBILITY OF MAPS IN FUZZY METRIC SPACE

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ABSTRACT

The aim of this paper to derive a new result for fuzzy metric space under the notion of semi compatible mappings.

KEYWORDS: Weakly Compatible, Semi Compatible Maps , Fixed Point, Implicit Relations and Fuzzy Metric Space

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INTRODUCTION

The study of common fixed point of mappings in a fuzzy metric space satisfying certain contractive conditions has been at the center of vigorous research activity. With the concept of fuzzy sets, the fuzzy metric space was introduced by Kramosil and Michalek[2]. In 1993, Jungck and Cho [3] introduced the concept of compatible mappings of type (A) by generalizing the definition of weakly uniformly contraction maps. Pathak and Khan [5] introduced the concept of compatible maps of type (A) and S-compatible by splitting the definition of compatible mapping of type(A). Jungck and Rhoades [4]termed a pair of self maps defined on a metric space to be coincidentally commuting or equivalently weakly compatible if they commute at their coincidence points. The aim of this paper to introduce a new concept of semi weakly compatible maps with a new class of functions implicit relations.

2. PRELIMINARY FOR SAKE OF CONVENIENCE WE RECALL SOME DEFINITIONS

Definition 2.1 A binary operation $*$: $[0, 1] \times [0, 1]\rightarrow [0, 1]$ is called a continuous t-norm if it satisfies the following conditions:

- $*$is associative and commutative.
- $*$is continuous.
- $a * 1 = a$, for all $a \in [0,1]$.
- $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$,

  for all a, b, c, d in [0, 1].

Definition 2.2 [38] A 3-tuple $(X,M,*)$ is called a fuzzy metric space if X is an arbitrary (non-empty) set, $*$ is continuous t-norm, and M is a Fuzzy set on $X^2 \times (0,\infty)$ satisfying the following conditions :

- $M(u,v, t) > 0$.
- $M(u, v, t) = 1$ if and only if $x = y$.

- $M(v, u, t) = M(v, u, t)$.
- $M(u, v, t) * M(v, w, s) \leq M(u, w, t + s)$
- $M(u, v, .) : (0,\infty) \rightarrow [0,1]$ is continuous
- $\lim_{n\rightarrow\infty} M(u,v,t) = 1$.

Note that it can be considered as the degree of nearness between x and y with respect to t. we identify u = v with M(u, v, t)= 1,for all t > 0.for all u, v, w ∈ X and s, t > 0.

Example 2.3. Let (X, d) be a metric space. Define a * b = ab (or a * b = min{a ,b}) for all x, y ∈ X and t > 0, M(x ,y ,t) = t /(t+ d(x, y)).

Then (X, M, *) is a fuzzy metric space and the fuzzy metric M induced by the metric d is often referred to as the standard fuzzy metric.

Lemma 2.4 Let (*X*, *M*, *) be a fuzzy metric space. Then for all x, y in X,M (x, y, *) is non-decreasing.

Lemma 2.5. Let (*X*, *M*, *) be a fuzzy metric space. If there exists $q \in (0, 1)$ such that $M(x, y, qt) \geq M(x, y, t)$ for all x ,y∈ X and t >0,then $x = y$.

Definition 2.6. A sequence $\{x_n\}$ in a fuzzy metric space (X, M, *) is called Cauchy sequence if $\lim_{n\rightarrow\infty} M(x_{n+p}, x_n,t) = 1$ for each t >0 and for each p > 0.

A fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

Lemma2.7.Let $\{x_n\}$ be a sequence in fuzzy metric space

(*X*, *M*, *)with condition $\lim_{n\rightarrow\infty} M(x ,y ,t)= 1$ .if there exists a number k ∈(0,1) Such that

$M (x_{n+1} , x_{n+2} , kt) \geq M (x_n , x_{n+1} , t)$, t > 0.

Then $\{x_n\}$ is a Cauchy sequence.

Proposition 2.8: In a fuzzy metric space (X,M,*) the limit of a sequence is unique

Definition 2.9: Two self mapping P and Q of a fuzzy metric space (X,M,*) are said to be compatible, if $\lim_{n\rightarrow\infty} M(PQx_n ,QPx_n ,t) = 1$ whenever $\{x_n\}$ is a sequence such that $\lim_{n\rightarrow\infty} Px_n = \lim_{n\rightarrow\infty} Qx_n = x$, for some x in X.

Definition 2.10: Let P and Q be maps from an FM-space(X,M, *) into itself. Then, the maps P and Q are said to be weakly compatible if they commute at their coincidence points, that is,

Px = Qx implies that PQx = QPx.

Remark 2.11: Every pair of compatible maps is weakly compatible but converse is not always true.

Definition 2.12: [100] Two self mapping P and Q of a fuzzy metric space (X,M,*) are said to be semi compatible ,$\lim_{n\rightarrow\infty} M(PQx_n, QPx,t) = 1$ whenever $\{x_n\}$is a sequence such that

$\lim_{n\rightarrow\infty} Px_n = \lim_{n\rightarrow\infty} Qx_n = p$, for some p in X.

It follows that if (P, Q) is semi-compatible and Py = Qy, then QPy = QPy (on taking $x_n = y$ for all n). Thus if the pair (P, Q) is semi-compatible, then it is weakly compatible, but the converse is not true always.

Proposition 2.13: [14,15]. Let P and Q be self-maps on a fuzzy metric space (X, M, *). If Q is continuous, then the pair (P,Q) is semi-compatible if and only if (P,Q) is compatible.

Proposition 2.14: Let P and Q be compatible and continuous self-maps on a fuzzy metric space (X,M,*). If there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Qx_n = x$, $x \in X$, where x is fixed point of either P or Q. Then P and Q are semi weakly compatible maps.

Definition 2.15: A class of implicit Relation. ϕ: $(R^+)^4 \to R$, non-decreasing in the first argument and satisfying the following conditions:

(ϕ_1) for $u, v \geq 0$, $\phi(u, v, u, v) \geq 0$ or $\phi(u, v, v, u) \geq 0$ implies that $u \geq v$.

(ϕ_2) $\phi(u, u, 1, 1) \geq 0$ implies that $u \geq 1$.

Example 2.16: Define $\phi(t_1,t_2,t_3,t_4) = at_1 + bt_2 + ct_3 + dt_4$, where a, b ,c, d are real constants. If a> max {b, d}and a+c =b+d > 0, then $\phi \in \Phi$.

MAIN RESULTS

Theorem 3.1: Let A, B, S, T, P and Q be self-mappings of a complete fuzzy metric space (X, M,*)satisfying:

(3.1.1) $A(X) \subset QT(X)$;$B(X) \subset PS(X)$;

(3.1.2) the pair (A, PS) is semi-compatible and (B, QT) is weakly compatible;

(3.1.3) one of A or PS is continuous;

(3.1.4)for some $\phi \in \Phi$,there exists $k \in (0,1)$ such that for all $x, y \in X$, and $t > 0$

$\phi(M(Ax, By, kt),M(PSx ,QTy,t),M(Ax, PSx,t),M(By,QTy,t)) \geq 0.$

(3.1.5) the pairs (P, S) and (Q, T) are commuting mappings;

(3.1.6) the pairs (P, A),(S, A),(Q, B) and (T, B) are semi weakly compatible mappings.

Then A, B, S, T, P and Q have unique common fixed point in X.

Proof: Let $x_0 \in X$ be any arbitrary point as $A(X) \subset QT(X)$ and

$B(X) \subset PS(X)$, there exist x_1; $x_2 \in X$ such that $Ax_0 = QTx_1$, $Bx_1 = PSx_2$.

Inductively, we can construct sequences $\{ y_n \}$ and $\{ x_n \}$ in X such that

$y_{2n+1} = Ax_{2n} = QTx_{2n+1}$

$y_{2n+2} = Bx_{2n+1} = PSx_{2n+2}$, for n = 0, 1, 2, . . .

Now using (3.1.4) with $x = x_{2n}$; $y = x_{2n+1}$, we get

$\phi(M(Ax_{2n},Bx_{2n+1}, kt),M(PSx_{2n},QTx_{2n+1},t), M(Ax_{2n}, PSx_{2n},t),M(Bx_{2n+1},QTx_{2n+1}, kt)) \geq 0$

that is

$\phi(M(y_{2n+1}; y_{2n+2}, kt),M(y_{2n}, y_{2n+1}, t),M(y_{2n+1}, y_{2n}, t),M(y_{2n+2}, y_{2n+1}, kt)) \geq 0,$

Using 2.1(i), we get $M(y_{2n+2}, y_{2n+1}, kt) \geq M(y_{2n+1}, y_{2n}, t)$.

Similarly by putting $x = x_{2n+2}$ and $y = x_{2n+1}$ in (3.1.5), we have

$M(y_{2n+3}, y_{2n+2}, kt) \geq M(y_{2n+1}, y_{2n+2}, t)$

$\phi\,(M(y_{2n+1}, y_{2n+2}, kt), M(y_{2n}, y_{2n+1}, t), M(y_{2n+1}, y_{2n}, t), M(y_{2n+2}, y_{2n+1}, kt)) \geq 0$

Using 2.1(i), we get $M(y_{2n+2}, y_{2n+1}, kt)\ \underline{\geq} M(y_{2n+1}, y_{2n}, t)$.

Similarly by putting $x = x_{2n+2}$ and $y = x_{2n+1}$ in (3.1.5), we have

$M(y_{2n+3}, y_{2n+2}, kt) \geq M(y_{2n+1}, y_{2n+2}, t)$.

Thus for any n and t, we have $M(y_n, y_{n+1}, kt) \geq M(y_{n-1}, y_n, t)$.

Hence by lemma 2.7, $\{ y_n \}$ is a Cauchy sequence in X. By the completeness of

X, $\{ y_n \}$ and its all subsequences $\{Ax_{2n}\}, \{Bx_{2n+1}\}, \{PSx_{2n}\}, \{QTx_{2n+1}\}$ are

also, converges to some point say $u \in X$.

Case I: Suppose PS is Continuous

then, we have $PSAx_{2n} \to PSu$, $(PS)^2x_{2n} \to PSu$. By semi-compatibility of the pair (A, PS) of maps,

we have $\lim_{n\to\infty} APSx_{2n} = PSu$.

Using (3.1.4) with $x = PSx_{2n}$,$y = x_{2n+1}$, we have

$\phi(M(APSx_{2n}, Bx_{2n+1}, kt), M((PS)^2x_{2n}, QTx_{2n+1}, t)(APSx_{2n}, (PS)^2x_{2n}, t),$

$M(Bx_{2n+1}, QTx_{2n+1}, kt)) \geq 0$

Letting $n \to\infty$ 1, we have $\phi\,(M(PSu , u, kt), M(PSu, u, t), 1, 1) \geq 0$.

As ϕ is non decreasing in the first argument, we have

$\phi\,(M(PSu, u, t), M(PSu, u, t), 1, 1) \geq 0$.

Using 2.1(ii), we get $M(PSu, u, t) \geq 1$, for all $t > 0$, which gives $PSu = u$.

Again by putting $x = u, y = x_{2n+1}$ in (3.1.4), we obtain

$\phi\,(M(Au, Bx_{2n+1}, kt), M(PSu, QTx_{2n+1}, t), M(Au, PSu, t), M(Bx_{2n+1}, QTx_{2n+1}, kt)) \geq 0$

Taking lim $n\to\infty$ and using 2.1(i), we get $u = Au$. Hence $Au = u = PSu$.

Since $A(X) \subset QT(X)$, there exists $w \in X$ such that $Au = PSu = u = QTw$.

By putting $x = x_{2n}$, $y = w$ in (3.1.4), we obtain

$\phi\,(M(Ax_{2n}, Bw, kt), M(PSx_{2n}, QTw, t), M(Ax_{2n}, PSx_{2n}, t), M(Bw, QTw, kt)) \geq 0$

Taking lim $n\to\infty$ and using 2.1(i), we get $u = Bw$.

Therefore $Bw = QTw = u$.

Since the pair (B, QT) is weakly compatible mappings, we get QTBw =BQTw,

that is Bu = QTu.

Now by putting x = y = u in (3.1.4) and using 2.1(ii), we have Bu = Au.

Therefore u = Au = PSu = Bu = QTu, that is, u is a common fixed point of the maps A, B, PS and QT.

Similarly it can be proved that if the map A is continuous then u is the common fixed point of the maps A, B, PS and QT

Uniqueness: Let z be another common fixed point of the maps A, B, PS and QT.

Putting x = u and y = z in (3.1.4) and using 2.1(i), we get

ϕ (M(Au,Bz, kt),M(PSu,QTz, t),M(Au, PSu, t),M(Bz,QTz, kt)) $\geq$ 0

that is ϕ (M(u, z, kt),M(u, z,t),1, 1) $\geq$0, yields that u = z.

Therefore u is the unique common fixed point of the self maps A, B, PS and QT in fuzzy metric space X.

From (3.1.6 and 3.1.7)), we have

Pz = P(PSz) = P(SPz) = (PS)Pz ; Pz = PAz = APz and Sz = S(PSz) =(SP)Sz = (PS)Sz;

Sz = SAz = ASz, implies that Pz and Sz are common fixed points of the maps PS and A.

CONCLUSIONS

Therefore z = Pz = Sz = Az = PSz. Similarly, Qz and Tz are common fixed points of the maps QT and B, therefore z = Qz = Tz = Bz = QTz. Hence zis the common fixed point of the maps A, B, S, T, P and Q. Further since z is the unique common fixed point of the maps A, B, PS and QT, consequently it is the unique common fixed point of the maps A, B, S, T, P and Q.

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